

# Classical and Quantum Intertwine

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## Abstract

Model interactions between classical and quantum systems are briefly discussed. These include: general measurement-like couplings, Stern-Gerlach experiment, model of a counter, quantum Zeno effect, SQUID-tank model.

## 1 Introduction

Heisenberg wrote in his book ‘Physics and Philosophy’ [1] that the Copenhagen interpretation of Quantum Theory rests on a paradox, namely the description of quantum phenomena in terms of classical concepts. We know that Quantum Mechanics works extremely well - it describes and computes (although we would not say: *explains*) not only those phenomena for which it was invented but also numerous others. But measurement-like processes cannot be described by Schrödinger equation. As emphasized by J. Bell [2]: ‘If, with Schrödinger, we reject extra variables, then we must allow that his equation is not always right’. Gisin and Percival formulated the same thesis as: ‘the Schrödinger equation is no longer the best for all practical purposes’ [3].

In a recent paper [4] we propose a mathematically consistent model of interaction between classical and quantum systems, which provides an answer to the question of how and why quantum phenomena become real as a result of interaction between

quantum and classical domains. Our results show that a simple dissipative time evolution can result in a dynamical exchange of information between classical and quantum levels of Nature. With a properly chosen initial state the quantum probabilities are exactly mirrored by the state of the classical system and moreover the state of the quantum subsystem converges for  $t \rightarrow +\infty$  to a limit which agrees with that required by von Neumann-Lüders standard quantum measurement projection postulate. In our model the quantum system is coupled to a classical recording device which will respond to its actual state. We thus give a *minimal mathematical semantics* to describe the measurement process in Quantum Mechanics. For this reason the toy model that we proposed can be seen as the elementary building block used by Nature in the communications that take place between the quantum and classical levels. The model has not only nice mathematical properties but it is also of great practical accessibility and, therefore, it is natural to formulate any practical problem by starting from the general structure of the ‘Ansatz’ we have proposed.

What is the fundamental difference between a classical and a quantum system? A purely quantum mechanical description of a cat would need about  $10^{27}$  independent parameters - and it would describe the cat as a closed system - thus certainly not the living cat. On the other hand to describe any relevant property of the cat we have to specify far fewer parameters. Having first developed a full quantum theory of a cat and ignoring after that all the degrees of freedom that we are not able to measure or that come from the environment is to allow for a classical behavior. Insisting on necessity of ‘quantising’ *all* the degrees of freedom can be compared to demanding that only letters of an alphabet should be used - but not the punctuation marks! Sure, printed matter would then look impressively homogeneous. But not sentence would ever end, no message would ever be transmitted. The Universe simply would not work that way.

In Section 2 we will briefly describe the mathematical and physical ingredients of the model. The purpose of Section 3 is to discuss the measurement process in this framework. The range of possible applications of the model is rather wide as will be shown in Section 4 with a discussion of the Zeno’s effect and in Section 5 with a description of the coupling between a SQUID and a damped classical oscillating circuit. Section 6 deals with some concluding remarks.

## 2 Interaction between a classical and a quantum system

Let us briefly describe the mathematical framework we will use. A good deal more can be said and we refer the reader to [4, 5]. Our aim is to describe a non trivial interaction between a quantum system  $\Sigma_q$  in interaction with a classical system  $\Sigma_c$ . To the quantum system there corresponds a Hilbert space  $\mathcal{H}_q$ . In  $\mathcal{H}_q$  we consider a family of orthonormal projectors  $e_i = e_i^* = e_i^2$ , ( $i = 1, \dots, n$ ),  $\sum_{i=1}^n e_i = 1$ , associated to an observable  $A = \sum_{i=1}^n \lambda_i e_i$ . The classical system is supposed to have

$m$  distinct pure states, and it is convenient to take  $m \geq n$ . The algebra  $\mathcal{A}_c$  of classical observables is in this case  $\mathcal{A}_c = \mathbb{C}^m$ . The set of classical states coincides with the space of probability measures. Using the notation  $X_c = \{s_0, \dots, s_{m-1}\}$ , a classical state is an  $m$ -tuple  $p = (p_0, \dots, p_{m-1})$ ,  $p_\alpha \geq 0$ ,  $\sum_{\alpha=0}^{m-1} p_\alpha = 1$ . The state  $s_0$  plays in some cases a distinguished role and can be viewed as the neutral initial state of a counter. The algebra of observables of the total system  $\mathcal{A}_{tot}$  is given by

$$\mathcal{A}_{tot} = \mathcal{A}_c \otimes L(\mathcal{H}_q) = \mathbb{C}^m \otimes L(\mathcal{H}_q) = \bigoplus_{\alpha=0}^{m-1} L(\mathcal{H}_q), \quad (1)$$

and it is convenient to realize  $\mathcal{A}_{tot}$  as an algebra of operators on an auxiliary Hilbert space  $\mathcal{H}_{tot} = \mathcal{H}_q \otimes \mathbb{C}^m = \bigoplus_{\alpha=0}^{m-1} \mathcal{H}_q$ .  $\mathcal{A}_{tot}$  is then isomorphic to the algebra of block diagonal  $m \times m$  matrices  $A = \text{diag}(a_0, a_1, \dots, a_{m-1})$  with  $a_\alpha \in L(\mathcal{H}_q)$ . States on  $\mathcal{A}_{tot}$  are represented by block diagonal matrices

$$\rho = \text{diag}(\rho_0, \rho_1, \dots, \rho_{m-1}) \quad (2)$$

where the  $\rho_\alpha$  are positive trace class operators in  $L(\mathcal{H}_q)$  with  $\sum_\alpha \text{Tr}(\rho_\alpha) = 1$ . By taking partial traces each state  $\rho$  projects on a ‘quantum state’  $\pi_q(\rho)$  and a ‘classical state’  $\pi_c(\rho)$  given respectively by

$$\pi_q(\rho) = \sum_{\alpha} \rho_{\alpha}, \quad (3)$$

$$\pi_c(\rho) = (\text{Tr}\rho_0, \text{Tr}\rho_1, \dots, \text{Tr}\rho_{m-1}). \quad (4)$$

The time evolution of the total system is given by a semi group  $\alpha^t = e^{tL}$  of positive maps <sup>1</sup> of  $\mathcal{A}_{tot}$  - preserving hermiticity, identity and positivity - with  $L$  of the form

$$L(A) = i[H, A] + \sum_{i=1}^n \left( V_i^* A V_i - \frac{1}{2} \{V_i^* V_i, A\} \right). \quad (5)$$

The  $V_i$  can be arbitrary linear operators in  $L(\mathcal{H}_{tot})$  such that  $\sum V_i^* V_i \in \mathcal{A}_{tot}$  and  $\sum V_i^* A V_i \in \mathcal{A}_{tot}$  whenever  $A \in \mathcal{A}_{tot}$ ,  $H$  is an arbitrary block-diagonal self adjoint operator  $H = \text{diag}(H_\alpha)$  in  $\mathcal{H}_{tot}$  and  $\{, \}$  denotes anticommutator i.e.

$$\{A, B\} \equiv AB + BA. \quad (6)$$

In order to couple the given quantum observable  $A = \sum_{i=1}^n \lambda_i e_i$  to the classical system, the  $V_i$  are chosen as tensor products  $V_i = \sqrt{\kappa} e_i \otimes \phi_i$ , where  $\phi_i$  act as transformations on classical (pure) states. Denoting  $\rho(t) = \alpha_t(\rho(0))$ , the time evolution of the states is given by the dual Liouville equation

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{i=1}^n \left( V_i \rho(t) V_i^* - \frac{1}{2} \{V_i^* V_i, \rho(t)\} \right), \quad (7)$$

where in general  $H$  and the  $V_i$  can explicitly depend on time.

**Remarks:**

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<sup>1</sup>In fact, the maps we use happen to be also *completely* positive.

1) It is possible to generalize this framework for the case where the observable  $A$  that is being measured admits a continuous spectrum (as for instance in a measurement of the position) with  $A = \int_{\mathbf{R}} \lambda dE(\lambda)$ . See [5, 6] for more details and Sections 3.3 and 5 where concrete examples of a situation of this type will be briefly described.

2) Since the center of the total algebra  $\mathcal{A}_{tot}$  is invariant under any automorphic unitary time evolution, the Hamiltonian part  $H$  of the Liouville operator is not directly involved in the process of transfer of information from the quantum subsystem to the classical one. Only the dissipative part can achieve such a transfer in a finite time.

### 3 The measurement process and all that

In [4] we propose a simple, purely dissipative Liouville operator (i.e. we put  $H = 0$ ) that describes an interaction of  $\Sigma_q$  and  $\Sigma_s$ , for which  $m = n + 1$  and  $V_i = e_i \otimes \phi_i$ , where  $\phi_i$  is the flip transformation of  $X_c$  transposing the neutral state  $s_0$  with  $s_i$ . We show that the Liouville equation can be solved explicitly for any initial state  $\rho(0)$  of the total system. Assume now that we are able to prepare at time  $t = 0$  the initial state of the total system  $\Sigma_{tot}$  as an uncorrelated product state  $\rho(0) = w \otimes P^\epsilon(0)$ ,  $P^\epsilon(0) = (p_0^\epsilon, p_1^\epsilon, \dots, p_n^\epsilon)$  as initial state of the classical system parametrized by  $\epsilon$ ,  $0 \leq \epsilon \leq 1$ :

$$p_0^\epsilon = 1 - \frac{n\epsilon}{n+1}, \quad (8)$$

$$p_i^\epsilon = \frac{\epsilon}{n+1}. \quad (9)$$

In other words for  $\epsilon = 0$  the classical system starts from the pure state  $P(0) = (1, 0, \dots, 0)$  while for  $\epsilon = 1$  it starts from the state  $P'(0) = (\frac{1}{n+1}, \frac{1}{n+1}, \dots, \frac{1}{n+1})$  of maximal entropy. Computing  $p_i(t) = \text{Tr}(\rho_i(t))$  and then the normalized distribution

$$\tilde{p}_i(t) = \frac{p_i(t)}{\sum_r p_r(t)} \quad (10)$$

with  $\rho(t) = (\rho_0(t), \rho_1(t), \dots, \rho_n(t))$  the state of the total system we get:

$$\tilde{p}_i(t) = q_i + \frac{\epsilon(1 - nq_i)}{\epsilon n + \frac{(1-\epsilon)(n+1)}{2}(1 - e^{-2t})}, \quad (11)$$

where we introduced the notation

$$q_i = \text{Tr}(e_i w), \quad (12)$$

for the initial quantum probabilities to be measured. For  $\epsilon = 0$  we have  $\tilde{p}_i(t) = q_i$  for all  $t > 0$ , which means that the quantum probabilities are exactly, and immediately after switching on of the interaction, mirrored by the state of the classical system. For  $\epsilon = 1$  we get  $\tilde{p}_i(t) = 1/n$ . The maximum entropy state is a stationary state of  $\Sigma_c$

and in this case we get no information at all about the quantum state by recording the time evolution of the classical one. For  $\epsilon = 0$ , that is when the measurement is exact, we get for the partial quantum state

$$\pi_q(\rho(t)) = \sum_i e_i w e_i + e^{-t} (w - \sum_i e_i w e_i),$$

so that

$$\pi_q(\rho(\infty)) = \sum_i e_i w e_i, \quad (13)$$

which means that the partial state of the quantum subsystem  $\pi_q(\rho(t))$  tends for  $t \rightarrow +\infty$  to a limit which coincides with the standard von Neumann-Lüders quantum measurement projection postulate.

### 3.1. Efficiency versus accuracy by measurement

Let us consider the case where

$$V_i = \sqrt{\kappa} e_i \otimes f_i, \quad (14)$$

$f_i$  being the transformation of  $X_c$  mapping  $s_0$  into  $s_i$ . In the Liouville equation we consider also an Hamiltonian part. The Liouville equation implies:

$$\dot{\rho}_0 = -i [H, \rho_0] - \kappa \rho_0, \quad (15)$$

$$\dot{\rho}_i = -i [H, \rho_i] + \kappa \rho_0 e_i, \quad (16)$$

where we allow for time dependence i.e.  $H = H(t)$ ,  $e_i = e_i(t)$ . Setting  $r_0(t) = \text{Tr}(\rho_0(t))$ ,  $r_i(t) = \text{Tr}(\rho_i(t))$ , and assuming that the initial state is of the form  $\rho = (\rho_0, 0, \dots, 0)$  we conclude that  $\dot{r}_0 = -\kappa r_0$  and thus  $r_0(t) = e^{-\kappa t}$  which implies that for small  $t$

$$\sum_{i=1}^n r_i(t) = 1 - e^{-\kappa t} \approx \kappa t, \quad (17)$$

from which it follows that a 50% efficiency requires  $\log 2/\kappa$  time of recording. It is easy to compute  $r_i(t)$  and

$$\tilde{p}_i(t) = \frac{r_i(t)}{\sum_{j=1}^n r_j(t)}$$

for small  $t$ . One obtains

$$\tilde{p}_i(t) = q_i + \frac{\kappa^2 t^2}{2} \frac{1}{\kappa} \left\langle \frac{de_i}{dt} \right\rangle_{\rho_0}, \quad (18)$$

where

$$\frac{de_i}{dt} \doteq \frac{\partial e_i}{\partial t} + i [H, e_i]. \quad (19)$$

Efficiency requires  $\kappa t \gg 1$  while accuracy is achieved if  $(\kappa t)^2 \ll \frac{\kappa}{\langle \dot{e}_i \rangle_{\rho_0}}$ . To monitor effectively *and* accurately fast processes we must therefore take at least  $\kappa \approx 10^2 \langle \dot{e}_i \rangle_{\rho_0}$ .

Suppose now that  $H$  and  $e_i$  do not depend on time. Then it is easy to show that if either  $\rho_0(0)$  or  $e_i$  commutes with  $H$ , we get  $\tilde{p}_i(t) = q_i$  exactly and instantly.

### 3.2. Stern Gerlach experiment

In the spirit of A. Böhm (cf. Ref. [7, Ch. XIII]) we model a Stern–Gerlach device by a pure spin 1/2 particle interacting with a spinless atom. Assuming that the magnetic field is linear in  $z$  the interaction Hamiltonian can be written

$$H_{int} = 2\mu_B B z \sigma_3. \quad (20)$$

Writing

$$\sigma_3 = \frac{|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|}{2},$$

$H_{int}$  is now given by

$$H_{int} = \mu_B B (|\uparrow\rangle\langle\uparrow| z - |\downarrow\rangle\langle\downarrow| (-z)). \quad (21)$$

Supposing now that the atom can be directly observed we can replace it for all practical purposes by a 3–state classical device ( $s_0, s_+, s_-$ ). The coupling is then modelled by

$$\sqrt{\kappa} (p \text{ flip}(0 \rightarrow +) + (1 - p) \text{ flip}(0 \rightarrow -)) \quad (22)$$

and we are now in position to approximate Stern–Gerlach experiment by our 3–state model. For more details see also [5].

### 3.3 Model of a counter

We consider a one-dimensional quantum mechanical particle. The counter sensitivity is described by an operator valued function  $f(t)$  and the quantum system  $\Sigma_q$  with  $\mathcal{H}_q = L^2(\mathbf{R}, dx)$  is coupled to a 2–state classical system. The Liouville equation for the state of the total system is

$$\dot{\rho} = -i [H, \rho] + V \rho V^* - \frac{1}{2} \{V^* V, \rho\}, \quad (23)$$

with

$$V = f \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & f \\ 0 & 0 \end{pmatrix} \quad (24)$$

Now explicitly we obtain

$$\dot{\rho}_0 = -i [H, \rho_0] - \frac{1}{2} \{f^* f, \rho_0\}, \quad (25)$$

$$\dot{\rho}_1 = -i [H, \rho_1] + f^* \rho_0 f. \quad (26)$$

Taking  $H = \frac{1}{i} \frac{d}{dx}$  and  $f = f^* = f(x, t)$  we obtain for the counting rate  $\dot{p}_1(t)$  in a free evolving state:

$$\dot{p}_1(t) = \int_{\mathbf{R}} |\Psi(x-t)|^2 f^2(x, t) e^{-\int_0^t f^2(x+s-t, s) ds} dx. \quad (27)$$

Assume now that we have to do with a point particle i.e.

$$|\Psi(x)|^2 = \delta(x - x_0)$$

we obtain in this idealized case

$$\dot{p}_1(t) = f^2(x_0 + t, t) e^{-\int_0^t f^2(x_0+s, s) ds} \quad (28)$$

which expresses the fact that the counting rate depends on how long the "detector" was already in contact with the particle. Let us remark that the quantum particle we consider is as in [8] an ultra-relativistic one. For more details see [5].

## 4 Quantum Zeno's Effect

The standard view of evolution of quantum states can be described as follows. Quantum states evolve through establishment of coherent superpositions. An initial state  $\Psi_0$  which is unstable develops into a superposition  $\Psi_t = a_0\Psi_0 + a_d\Psi_d$  of undecayed and decay-product states. A measurement of the survival probability  $|\langle\Psi_t, \Psi_0\rangle|^2$  projects the state  $\Psi_t$  back to the initial undecayed state; repeated, frequent measurements can inhibit or even prevent the decay. This is called the Quantum Zeno effect. W. Yourgrau [9] attributed it to A.M. Turing while B. Misra and E.C.G. Sudarshan [10] coined the name and discussed the relevant timescales at work. Using our model of a continuous measurement we can easily discuss this effect for a quantum spin 1/2 system coupled to a 2-state classical system [11]. We consider only one orthogonal projector  $e$  on the Hilbert space  $\mathcal{H}_q = \mathbb{C}^2$ . To specify the dynamics we choose the coupling operator  $V$  in the following symmetric way:

$$X = \sqrt{\kappa} \begin{pmatrix} 0 & e \\ e & 0 \end{pmatrix}. \quad (29)$$

The Liouville equation for the total state  $\rho = \text{diag}(\rho_0, \rho_1)$  reads now:

$$\dot{\rho}_0 = -i[H, \rho_0] + \kappa(e\rho_1e - \frac{1}{2}\{e, \rho_0\}), \quad (30)$$

$$\dot{\rho}_1 = -i[H, \rho_1] + \kappa(e\rho_0e - \frac{1}{2}\{e, \rho_1\}). \quad (31)$$

The partial quantum state  $\pi_q(\rho) = \hat{\rho} = \rho_0(t) + \rho_1(t)$  evolves in this particular model independently of the state of the classical system, which expresses the fact that we

have here only transport of information from  $\Sigma_q$  to  $\Sigma_c$ . The time evolution of  $\hat{\rho}(t)$  is given by

$$\dot{\hat{\rho}} = -i [H, \hat{\rho}] + \kappa(e\hat{\rho}e - \frac{1}{2}\{e, \hat{\rho}\}) \quad (32)$$

Let us now choose the Hamiltonian part  $H = \frac{\omega}{2}\sigma_3$  and  $e = \frac{1}{2}(\sigma_0 + \sigma_1)$ , and to start with the quantum system  $\Sigma_q$  being for  $t = 0$  in the eigenstate of  $\sigma_1$ . We repeatedly check with frequency  $\kappa$  if the system is still in this initial state, each ‘yes’ inducing a flip in the coupled classical device, which we continuously observe. The solution of (32) such that  $\hat{\rho}(0) = e$  can be easily found. Moreover it is possible for strongly coupled system i.e. for  $\kappa t \gg 1$  and  $\kappa/\omega \gg 1$  to obtain asymptotic formulae for the distance travelled by the quantum state  $d(\hat{\rho}(t), e)$  in the Bures or in the Frobenius norm  $\|\hat{\rho}\|^2 = \text{Tr}(\hat{\rho}^2)$ . In this asymptotic regime we can show that the Bures distance achieved during the coupling is given by

$$d(\hat{\rho}(t), e) \approx \omega\sqrt{t/\kappa}. \quad (33)$$

The effect of slowing down the evolution of the quantum system can be confirmed by an independent, strong but non-demolishing, coupling of a third classical device. In [12] we show moreover that a piecewise deterministic Markov process taking values on pure states of the total system is naturally associated to the Liouville equation.

#### 4.1. Comments on ‘meaning of the wave function’

One could think of using the Quantum Zeno effect for slowing down the time evolution, so that the state of a quantum system can be determined by measurements of sufficiently many observables. This idea, however, would not work, similarly like would not work the idea of ‘protective measurements’ of Y. Aharonov et. al. (cf. Refs [13, 14]). To apply Zeno-type measurement, similarly like to apply a ‘protective measurement’ one would have to know the state beforehand. This negative statement does not mean that the quantum state cannot be determined. To the contrary, by coupling a quantum system to a 3-state asymmetric (but *not* symmetric!) device according to our recipe (cf. [5] for more details), and using sufficiently strong and sufficiently short couplings, one can measure an arbitrary finite number of expectation values of quantum observables, without disturbing the quantum system beyond prescribed limits. Far from being able to determine the quantum state with an *infinite precision*, such a, theoretically feasible, sequence of measurements should be sufficient for all practical purposes.<sup>2</sup> Moreover, it is natural to try to use the progressive, incomplete, knowledge of the quantum state for slowing down the evolution during next series of measurements. This last idea needs however further investigations.

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<sup>2</sup> For some quantum system there will be, of course, unsurmountable technical difficulties

## 5 SQUID coupled to a damped classical oscillator

In a superconducting ring of uniform thickness the quantized flux states do not interact. The quantum state of the ring can only be changed by warming it up, changing the applied external field and then cooling it down to its superconducting temperature again. However, mixing of the flux states becomes possible if the ring has a so-called weak link, across which the magnetic flux can leak. These objects are called SQUIDS and possess a wide variety of macroscopic quantum mechanical properties. In recent years there has been considerable discussion of the dynamics of a model quantum-mechanical system consisting of a SQUID coupled to a dissipative classical linear oscillator [15, 16, 17, 18]. Our aim is to briefly show that our framework is very well adapted to give a description of a ‘mini’ SQUID and to discuss the behaviour of the coupled system consisting of a macroscopic classical system (tank circuit) and a single quantum object (SQUID). The states of the total system are now described by  $\rho_t(\phi, \pi)$ , with  $\rho_t(\phi, \pi) \geq 0$  and

$$\int_{\mathbf{R}^2} \text{Tr}(\rho_t(\phi, \pi)) d\phi d\pi = 1.$$

The Liouville equation that describes SQUID+tank circuit is:

$$\begin{aligned} \frac{\partial \rho(\phi, \pi, t)}{\partial t} = & -\frac{i}{\hbar} [H(\phi), \rho(\phi, \pi, t)] - \pi \frac{\partial \rho}{\partial t} + \\ & + \left( \frac{\pi}{RC} + \frac{\phi}{LC} - \frac{1}{C} I_{IN}(t) \right) \frac{\partial \rho}{\partial \pi} + \\ & + \frac{\rho}{RC} + (L\rho)(\phi, \pi, t), \end{aligned} \quad (34)$$

where

$$\begin{aligned} L\rho(\phi, \pi, t) = & \alpha \int_{\mathbf{R}} f(\Phi - \Phi_{EXT} - a) \rho(\phi, \pi - a, t) f(\Phi - \Phi_{EXT} - a) da \\ & - \alpha \beta \rho(\phi, \pi, t), \end{aligned} \quad (35)$$

$f(x) = f(-x) \geq 0$  being a real function characterising the coupling (e.g. a Gaussian),  $\beta = \int f(x)^2 dx$ , and  $\alpha\beta = \mu/(C\Lambda)$ , with  $R, C, L$  characterizing electric properties of the tank circuit,  $\mu$  the mutual SQUID-tank inductance while  $\Lambda$  the superconducting ring geometry. The SQUID Hamiltonian is defined on the Hilbert space  $\mathcal{H}_q = L^2(\mathbf{R}, d\Phi)$

$$H(\phi) = \frac{Q}{2C} + \frac{(\Phi - \Phi_{EXT})}{2\Lambda} - \hbar\omega \cos\left(\frac{\Phi}{\Phi_0}\right), \quad (36)$$

$$\Phi_{EXT} = \phi_{ext} + \mu\phi, \quad (37)$$

$$Q = -i\hbar \frac{d}{d\Phi}, \quad (38)$$

$$\Phi_0 = \frac{\hbar}{2e}. \quad (39)$$

Notice that the SQUID Hamiltonian depends explicitly on the classical parameter  $\phi$ . It can be now shown (see [5]) that the expectation value of  $\phi$  satisfies the non-linear equation postulated and investigated in Refs. [17, 18]. The same method that we used here for modelling of SQUID-tank coupling can be applied as well to other problems where the classical system is expected to respond to ‘averages’ of some quantum observables. <sup>3</sup>

## 6 Concluding remarks

The exceptionally brilliant calculational successes of Quantum Mechanics cannot cause to forget the degree of conceptual confusion still present. The essential problem follows from the fact that Quantum Mechanics is the most fundamental theory we know. But if it is really fundamental, it should be universally applicable. In particular quantum physics should be able to explain also the properties of macroscopic objects and occurrence of macroscopic events. But measurement situations show clearly that it is impossible to apply standard Quantum Mechanics in a consistent way to all relevant situations. If there is such a universal theory, it is therefore not Quantum Theory.

Indeterminism is an implicit part of classical physics. Irreversible laws are fundamental and reversibility is an approximation. We cannot refrain quoting from R. Haag’s paper *Irreversibility introduced on a fundamental level* [19]: ‘... once one accepts indeterminism, there is no reason against including irreversibility as part of the fundamental laws of Nature. ’We propose to consider  $\Sigma_{tot} = \Sigma_q \otimes \Sigma_c$  and the behaviour associated to the total algebra of observables  $\mathcal{A}_{tot} = \otimes \mathcal{A}_q \otimes \mathcal{A}_c = C(X_c) \otimes L(\mathcal{H}_q)$  is now taken as the fundamental reality with *pure* quantum behaviour as an approximation valid in the exceptional cases when dissipative effects can be neglected. In  $\mathcal{A}_{tot}$  we can describe irreversible changes occurring in the physical world – like the formation of a track in bubble chamber or the tragical cat’s death – as well as (idealized) reversible pure quantum processes.

Lot of work must still be done, lot of prejudices overcome. What we propose does not aspire to be a magic medicine that will rejuvenate Quantum Theory and make it Universal-And-True-For-Ever. But, perhaps, it will help to stop the bleeding from some open scars..

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<sup>3</sup>Like, for instance, classical gravitational field is expected to have as its source averaged energy-momentum tensor of quantized matter.

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